Mathematics - Course 221

APPENDIX 1: REVIEW EXERCISES

Review Exercise #1: Reliability

1. A system of 12 dousing valves, tested monthly, has developed 10 failures of individual valves in 8 years' operation. Calculate the unreliability of an individual valve.

2. Calculate the annual risk of a nuclear incident at a reactor, which, during 9 years' operation, developed the following faults:

   (a) 3 unsafe failures of the regulating system.

   (b) 50 complete failures of the protective system, failures of which are detected and corrected at the beginning of each shift.

3. At a certain nuclear generating station, three independent divisions of equipment protect against nuclear accidents:

   (i) process equipment with a failure frequency of 0.3 per annum,

   (ii) protective equipment with unreliability of 2×10^{-3}, and

   (iii) containment equipment with unreliability of 5×10^{-3}.

   Calculate the annual risk (frequency) of

   (a) an incident consisting of process failure combined with simultaneous failure of either protective or containment systems.

   (b) simultaneous failure of all three systems.

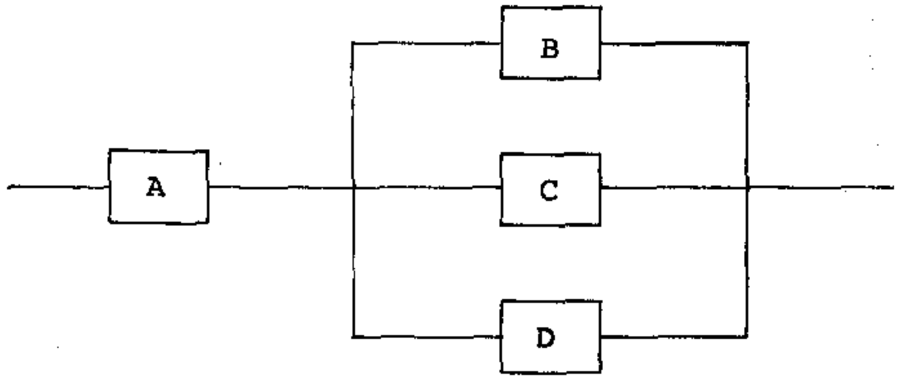
4. Monthly testing of 6 safety switches has revealed 8 failures of individual switches during 15 years' operation.

   (a) Calculate the unreliability of a switch.

   (b) How, without altering the equipment, could the unreliability in (a) be decreased by a factor of about 30?

4. (c) How often should the switches be tested if the permitted unreliability of a switch is 10^{-2}?

5.



In the above system, a system failure consists of a failure of either component A, or a failure of at least two of B, C, D.

Calculate the unreliability of the system, given component reliabilities,

$$Q_A = 0.05, \text{ and}$$
$$Q_B = Q_C = Q_D = 0.1.$$

6. A pump designed for continuous operation has failed 6 times in 5 years' operation, with total down time of 124 hours. Calculate the unavailability of

(a) the pump

(b) a system of three such pumps in a 3 x 50% parallel arrangement.

Review Exercise #2 (Lessons 221.20-1, 421.40-2, 321.10-3)

1. Find the equation of the following lines in the xy-plane:

   (a) passing through (2,4) and (3,1)
   (b) passing through (-4,5) with slope $-\frac{4}{3}$
   (c) with y-intercept -5 and slope $\frac{2}{5}$
   (d) passing through (4,-1) and parallel to $2x - 5y + 8 = 0$
   (e) passing through (6,-2) and perpendicular to $5x + 3y - 2 = 0$
   (f) passing through (-5,-3) with angle of inclination 135°

2. Given $f(x) = x^2 - 2x + 5$ evaluate
   (a) $f(0)$
   (b) $f(-5)$
   (c) $f(x + a)$

3. Plot $y = x^2 - 2x + 5$ and label the roots on the graph.

4. Find the slope and y-intercept of the following lines:

   (a) $6x - 5y + 8 = 0$
   (b) $\frac{2}{15}x + \frac{4}{3}y - 1 = 0$

5. For each line in Question 4 state the change in

   (a) x corresponding to an increase of y by 2
   (b) y corresponding to an increase of x by 5
   (c) y corresponding to a decrease in x of $\frac{1}{2}$

Review Exercise #3 (Lessons 221.20-2, 3 and 321.10-4)

1. If $s(t) = t^2 + 2t$

   (a) plot s(t) vs t

   (b) find the average velocity over the first 2 seconds
   ($t = 0$ to $t = 2$)

   (c) find the formula for the instantaneous velocity, v(t), at time t, using

$$v(t) = \lim_{\Delta t \to o} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

   (d) find v(2). (Compare with (b))

2. Find v(t) and a(t), using differentiation formulas if

   (a) $s(t) = -2\sqrt{t}$ (find v(4))

   (b) $s(t) = t^3 - 4t$ (find v(5))

3. Differentiate with respect to x:

   (a) $5x^3 - 2x + 13$

   (b) $4\sqrt[3]{x^2}$

   (c) $-x^2 + \frac{2}{x^2}$

   (d) $\frac{x^2 - x}{\sqrt{x}}$

4. Find equation of tangent and normal to curve $y = x^2 - 2x + 3$
   at (a) $x = -1$ (b) $x = 1$

5. The decay constant λ for a radionuclide is $0.010 s^{-1}$

   (a) Find the activity of a 10 curie source after 2.0 minutes.

   (b) There are 3.7×10^{13} radioactive nuclei to start with. How many remain after 2 minutes?

6. Given that a radioactive source has decay constant
   $\lambda = 1.2 \times 10^{-5}\ s^{-1}$, find the

   (a) activity of a 10 Ci source after 20 hours

   (b) half-life of the source

   (c) time for a 10 Ci source to decay to 1 Ci

   (d) number of radioactive nuclei in a 10 Ci source

Review Exercise #4: Practice in Solving Quadratics

1. At what points is 2 the slope of the curve

   $$y = \frac{x^3}{3} + x^2 - 13x + 10?$$

2. At what instants does the velocity equal zero if

   $$s(t) = 2t^3 + \frac{11t^2}{2} - 10t - 5?$$

3. At what x values are the slopes of the two curves equal?

   (a) $y_1 = x^3; \quad y_2 = x^2 + x$

   (b) $y_1 = 3x^3 + \frac{19}{2}x^2; \; y_2 = 12 + 15x - \frac{x^3}{3}$

4. At what instants are the velocities equal for the following displacement functions?

   $$s_1(t) = t^3 + \frac{5}{2}t^2 - 3t \; ; \quad s_2(t) = \frac{t^3}{3} + 3t^2 + 4$$

Review Exercise #5: (To end of lesson 221.20-4)

1. Using data tables, find $\ell n\ x$ for x = .001, .01, .1, .5, 1 2, 4, 6, 8, 10. Use your table of values to plot a graph of $\ell n\ x$ versus x.

2. Find (a) $\ell n\ e^{2.5}$ (b) $\log \frac{1}{10,000}$

   (c) $\log 10^{4/t}$ (d) $\ell n\ e^{-\lambda t}$

3. If $A(t) = A_0 e^{-\lambda t}$ prove that half-life $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

4. The decay constant for a radionuclide is $3.8 \times 10^{-5}\ s^{-1}$. Find the

   (a) half-life

   (b) number of half-lives to decay from 8 Ci to 1 Ci.

   (c) time for activity to die down to 1% original

   (d) time for activity to die from 2 Ci to 100 μCi

   (e) number of radioactive nuclei in a 2 Ci source
   (1 Ci = 3.7×10^{10} dps)

5. Given $\Delta k = .008$, $L = 0.25$, $P_0 = 10$ watts, $P(t) = P_0 e^{\frac{\Delta k}{L}t}$

   (a) find $P(t)$, $P^1(t)$ at t = 20 minutes

   (b) If rated power were 1000 MW, find the time to reach rated power.

6. Given $P(t) = P_0 e^{\frac{\Delta k}{L}t}$ show that rate log power is directly proportional to $\frac{\Delta k}{L}$. 'Explain the importance of a rate log meter to reactor operation.

7. Differentiate with respect to variable x or t as applicable:

   (a) $\frac{6}{\sqrt{x}}$ (b) $2x^3 - 11x^2 + 14$ (c) $-\sqrt{x} + \frac{2}{x}$

   (d) e^{2x^2} (e) $100e^{-4t}$ (f) $e^{2/t}$

Review Exercise #6: (To end of lesson 221.20-4)

1. Differentiate: (a) x^{15} (b) $\frac{-2}{\sqrt{x}}$ (c) $\frac{x^2-x}{\sqrt[3]{x}}$

(d) $e^{\sqrt{x}}$ (e) e^{-1/x^2}

2. Find v(t) and a(t) given

(a) $s(t) = 50t - 9.8t^2$ (b) $s(t) = e^t + t$

(c) $s(t) = 2t^3 - 14t^2 + 5t - 8$

3. Find equation of tangent and normal to $y = x^2 - 6x - 16$ for

(a) $x = -1$ (b) $x = 3$

4. Find equation of the line

(a) through (-2,5) and (6,-1)

(b) through (4,1) and having y-intercept of -3.

5. Find slope and y-intercept of $4x + 5y - 13 = 0$.

6. Find the roots of $f(x) = x^2 - 6x - 16$.

7. Given $\Delta k = .005$, $L = .12$ s, $P_0 = 50$ W, find, using $P(t) = P_0 e^{\frac{\Delta k}{L}t}$,

(a) reactor power after 2 minutes

(b) time for reactor to gain from 1% to 100% power.

8. Find (a) $\ln e^{-t^2}$ (b) $\log 10^{\sqrt[3]{t}}$

9. Given $t_{\frac{1}{2}} = 10$ minutes, find

(a) decay constant (b) time for source to decay from 0.5 Ci to 10 μCi

10. Find the activity of a source consisting of 2.0×10^{15} radioactive atoms if the decay constant is 6.5×10^{-4} s^{-1} in

(a) dps (b) curies

Review Exercise #7: Integration Problems (Lesson 221.30-1)

1. Find the area under the following curves between the indicated limits:

   (a) $y = 2x + 1$, $x = 0$ to $x = 5$

   (b) $y = \sqrt{x}$, $x = 4$ to $x = 9$

   (c) $y = \sqrt[3]{x^2} + 3x + 2$, $x = 8$ to $x = 27$

2. Find (i) $v(t)$ (ii) $s(t)$ given the following:

   (a) $a(t) = -2$, $v(0) = 6$, $s(0) = 0$

   (b) $a(t) = 2\sqrt{t}$, $v(0) = 0$, $s(0) = 100$

   (c) $a(t) = -t + 3$, $v(0) = v_0$, $s(0) = 0$

3. (a) Given $\frac{dy}{dx} = 4x + 5$ find y

   (b) Given $\frac{ds}{dt} = t^{3/2}$ find $s(t)$

   (c) Given $\frac{dv}{dt} = 6t$ find $v(t)$

4. Find $v(t)$ and $s(t)$ given

   (a) $a(t) = -9.8\ m/s^2$ $v(0) = v_0$, $s(0) = 0$

   (b) $a(t) = 0\ m/s^2$ $v(0) = v_0$, $s(0) = 0$

   (c) $a(t) = \sqrt[3]{t}\ m/s^2$ $v(0) = v_0$, $s(0) = 10$ m

5. Integrate:

   (a) $x^2 - 2$ (b) $2t^3 - 4t$

   (c) $\sqrt{x}$ (d) t^{-5}

   (d) $\frac{2}{x^2}$ (f) $\frac{5}{\sqrt{x}} + 14$

6. Find the displacement function $s(t)$ if the velocity function is

   (a) $v(t) = 2t - 3$

   (b) $v(t) = 3\sqrt{t} + 4$

7. Find $v(t)$ and $s(t)$, given

   (a) $a(t) = -5\ m/s^2$, $v(0) = 10$ m/s, $s(0) = 0$

   (b) $a(t) = 2t^2$, $v(0) = 0$, $s(0) = 0$

<u>Review Exercise #8: (To end of lesson 221.30-1)</u>

1. Given $f(x) = x^3 + x^2 - 17x + 15$. Graph $y = f(x)$, and label the roots of $f(x) = 0$.

2. Find the roots of $f^1(x) = 0$, given $f(x)$ as in Question #1. What is the significance of <u>these</u> roots to the curve $y = f(x)$?

3. Find the equation of the tangent and normal to $y = f(x)$ of Question #1 at $x = 1$.

4. Evaluate

   (a) $e^{\ln 0.1}$ (b) $10^{\log t^2}$ (c) $\ln e^{-2/t}$

   (d) $\log 10^y$ (e) $-\ln e^{-0.4}$ (f) $10^{\log 100}$ (g) $e^{\ln \lambda t}$

5. A source consisting of 8.6×10^{13} radioactive atoms is decaying at the rate of 7.5×10^9 dps. Find

   (a) the decay constant
   (b) the half-life
   (c) the time required for the activity to die to 1 μCi.

6. If the half-life of a radionuclide is 8.4 minutes, find

   (a) the decay constant

   (b) the time for source activity to decrease by a factor of 1000.

7. Differentiate:

   (a) $x^7 - 6x^3 + \sqrt[3]{x}$ (b) $\dfrac{x^3 - 1}{\sqrt{x}}$

   (c) $\sqrt[3]{x^2}$ (d) $x^{2/5} + \dfrac{a}{x}$

   (e) e^{-2/t^2} (f) e^{x^2-4}

   (g) $\sqrt{x}(x^3 - \dfrac{1}{x})$

8. If reactor power builds up from 100 W to 1000 MW is 5.0 minutes and the mean time between neutron generations is .1 seconds, find the reactivity.

9. If $N(t) = N_0e^{-\lambda t}$, prove that $\frac{dN}{dt} = -\lambda N$.

10. Find the equation of the line

    (a) parallel to $5y - 2x + 3 = 0$ and passing through the origin

    (b) perpendicular to $5y - 2x + 3 = 0$, and having the same y-intercept

    (c) having an angle of inclination of 45° and the same x-intercept as $5y - 2x + 3 = 0$.

11. (a) Given $f^1(x) = -2x + 0.4$, find $f(x)$ if $f(0) = -7$.

    (b) Given acceleration $a(t) = \frac{2}{\sqrt{t}}$, $v(0) = 1$, $s(0) = 4$, find $v(t)$, $s(t)$.

    (c) The R/C of $g(x)$ with respect to x is $-\frac{a}{x^2} - 10$. Find $g(x)$.

    (d) y increases 3 times as fast as x. If $y = -5$ when $x = 0$, find y as a function of x.

12. Find the area under the curve

    (a) $y = x^3$ from $x = 1.5$ to $x = 5$

    (b) $y = e^x$ from $x = 0$ to $x = 3$

    (c) $y = \sqrt[3]{x}$ from $x = 1$ to 8

Review Exercise #9: (To end of Lesson 221.30-2)

1. Plot $y = 1.5^x$, $-5 \leq x \leq 10$.

2. Plot A vs t where $A(t) = A_0 e^{-\lambda t}$, $A_0 = 100$ Ci, and $\lambda = 0.01\ s^{-1}$

   (a) using semi-log graph paper for $10 \leq t \leq 1000$ s.

   (b) using linear graph paper for $0 \leq t \leq 1000$ s.

   (Note the advantages/disadvantages of logarithmic graph paper.)

3. The force F to extend a spring varies directly as the extension x in meters.

   ie, $F = kx$, where k is called the spring constant

   (a) Prove that the work done in stretching the spring x meters equals $\frac{1}{2}kx^2$.

   (b) How much work is done in stretching a spring by 0.25 m if its spring constant is 1.2×10^4 N/m?

4. The force of gravity, F_g, on a satellite of mass M_s varies inversely as the square of its distance x from the earth's centre, ie,

$$F_g = \frac{GM_eM_s}{x^2}$$

   where G is the universal gravitation constant, and M_e is the earth's mass.

   (a) Prove that the work done by a rocket to lift a satellite d meters above the earth's surface (neglecting friction) is

$$W = \frac{GM_eM_sd}{R_e(R_e + d)}$$

   where R_e is the earth's radius.

   (b) How much energy must the rocket provide to free the satellite from earth's gravity altogether?

5. Translate the following rate-of-change statements to differential equations:

   (a) The torque T on a wheel equals the product of the wheel's moment of inertia I times the time rate of change of the wheel's angular velocity, ω.

(b) The voltage V across an inductor equals the product of the inductance L times the rate of change of the current i with respect to time.

6. For a poison-injection shut down of a reactor, gadolineum (Gd) is injected into the moderator at a concentration of 20 mg Gd/kg D_2O. This Gd must be removed before the reactor can be restarted. Moderator cleanup is achieved by cycling the moderator through ion exchange columns, and the concentration C(t) of Gd remaining in the moderator after t hours is given by the expression.

$$C(t) = 20\ e^{-0.35t}\ \text{mg Gd/kg } D_2O$$

Find:

(a) the time required to reduce the concentration to 0.8 mg Gd/kg D_2O, at which the reactor can be restarted.

(b) an expression for the <u>rate</u> at which Gd is removed, as a function of time.

(c) the total reduction in Gd concentration during the first 10 hours.

(d) the average rate of Gd concentration during the first 10 hours

(e) the instantaneous rate of Gd removal at half-time (t = 5 h). Why is this rate different from that of (d)?

(f) the average concentration during the first 10 hours.

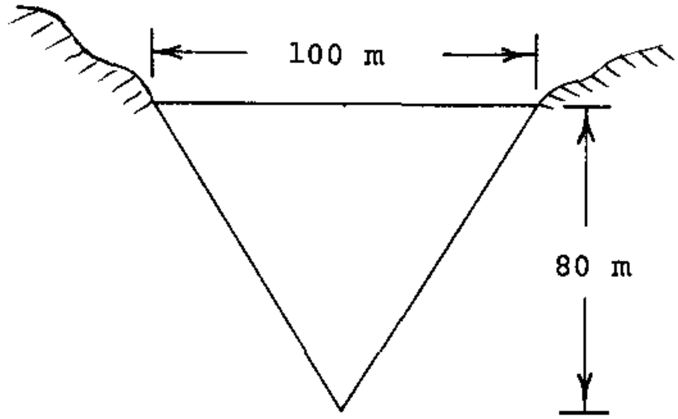
7. For the V-shaped hydraulic dam illustrated below, assuming water level coincides with top of dam, find:

(a) the total force exerted by the water against the face of the dam

(b) the average pressure of the water against the dam face

(c) the center of pressure exerted by the water.

7.



L.C. Haacke